the last term in the table: it will therefore resolve into square numbers any odd number up to $9121+190=9211$.

With reference to the mode in which the intervals in the table may be filled up, the author states the following general theorems relating to the sums of three square numbers, by means of which the roots may be varied, and yet the sum of the squares remain the same.

Theorem D.—If any three terms of an arithmetical series, and omitting the 4th term, the three following terms be arranged thus,

$$\begin{array}{ccc} a+b, & a+2b, & a+6b, \\ a\ \ , & a+4b, & a+5b, \end{array}$$

the sum of the squares of each set of terms will be the same.

Theorem E.—If four numbers in arithmetical progression be placed thus,

$$\begin{array}{cc} a\ \ , & a+2b, \\ a+4b, & a+6b, \end{array}$$

and the sum of the 1st and 4th be divided into two parts whose difference shall be four times the arithmetic ratio, as $a+7b-(a-b)$, and the parts be placed with the terms, the greater with the less, and the less with the greater, thus,

$$\begin{array}{ccc} a\ \ , & a+2b, & a+7b, \\ a-b, & a+4b, & a+6b, \end{array}$$

the sum of the squares will be equal.

Theorem F.—Let two numbers which differ by $2n$ be placed thus:

$$\begin{array}{cc} a+n, & a+n, \\ a-n, & a-n, \end{array}$$

then if the sum of the four $(4a)$ be divided so as to have the same difference $(2n)$, and the parts be placed, the less with the greater, and the greater with the less, thus,

$$\begin{array}{ccc} a+n, & a+n, & 2a-n, \\ a-n, & a-n, & 2a+n, \end{array}$$

the sum of the squares shall be the same.

The author illustrates this part of the subject by deducing six forms of roots whose squares $=197$.

January 12, 1854.

The LORD CHIEF BARON, V.P., in the Chair.

Commander Kay, R.N., was admitted into the Society.

A paper was read, entitled "On some New and Simple Methods of detecting Manganese in Natural and Artificial Compounds, and

of obtaining its Combinations for œconomical or other uses." By Edmund Davy, Esq., F.R.S., M.R.I.A. &c. Received December 4, 1853.

In this paper the growing importance of manganese since its discovery, and its extensive distribution in Nature are noticed. Manganese is chiefly found combined with oxygen, but its oxides are commonly mixed with those of iron, and though different methods of separating them have been recommended, yet no very simple or unobjectionable test for manganese seems to be known. Two methods for detecting manganese are recommended, viz.—

1. The pure hydrated fixed alkalies, potash and soda, and especially potash. 2. Sulphur.

With regard to the first method. Though the compound *Chameleon mineral* made by strongly heating nitre or potash and peroxide of manganese together, has long been known, yet it appears hitherto to have escaped observation, that potash seems to be a more delicate test of manganese than any other known substance. The use of potash in this way is simple and easy; it is employed in solution; equal weights of the alkali and water form a fluid well-adapted for the purpose; different metals may be used in the form of slips on which to make the experiments, but a preference is given to silver foil, as it is less acted on by alkalies than platina, and is more readily cleaned. A slip of such foil, about two or three inches in length and half an inch wide, answers well. Solids, to be examined for manganese, are finely pulverized; fluids require no preparation; the smallest portion of either is mixed with a drop or part of a drop of the alkali on the foil and heated by a spirit-lamp (for many experiments a candle affords sufficient heat), when on boiling the alkali to dryness and raising the heat, the characteristic green manganate of potash will appear on the foil. The delicacy of the alkali as a test thus applied, will be obvious on using the most minute portions of manganese ores in fine powder, and the author's son, Dr. E. W. Davy, readily detected manganese in a single drop of a solution containing one grain of solid sulphate in ten thousand grains of water. The presence of other oxides do not appear to impair the efficacy of this test. A strong solution of hydrate of soda in water, used in a similar manner, affords an excellent test for manganese, little inferior in delicacy to potash, but the latter is shown to be preferable.

Carbonate of soda has long been regarded as one of the most delicate tests of manganese, especially if aided by a little nitrate or chlorate of potash, but that carbonate is much inferior as a test for manganese to potash or soda, requiring a far higher temperature to form the manganate of soda, and the aid of oxidizing substances, as nitre and chlorate of potash, which are quite unnecessary with those alkalies. Borax, too, in point of delicacy is not to be compared with the fixed alkalies as a test for manganese.

The author is of opinion that the fixed alkalies in solution and silver foil will form a valuable addition to the agents employed by the mineralogist and chemist in the examination of minerals, ores, &c.

2. *Sulphur.*—If a little flowers of sulphur be mixed with about its own bulk of the common peroxide of manganese, and exposed on a slip of platinum foil to a red heat, sesquioxide, sulphuret and sulphate of manganese will be formed, and by continuing the heat for a short time, an additional quantity of the sulphate will be produced from the sulphuret. On treating the mass with water and filtering the fluid, a solution of sulphate of manganese will be obtained which will yield a white precipitate with the ferrocyanide of potassium, without a trace of iron.

Similar experiments may be made with any manganese ores, or with substances known or suspected to contain manganese. The quantity of materials operated on may be increased or diminished at pleasure; but if increased, the heat should be continued a little longer, to decompose any remaining sulphuret, and thus add to the quantity of sulphate formed. In the same way manganese was detected in some minerals in which it was known to exist, and in others in which it had not been previously found; likewise in soils and subsoils, in the ashes of coal and peat, in a number of pigments, and also in the ashes of different fabrics partially dyed brown by manganese.

Sulphate of manganese is formed, with sulphuret, when sulphurous acid gas is made by heating a mixture of peroxide of manganese and flowers of sulphur, even in close vessels. The sulphate may also be more readily obtained, in quantity, by simply boiling a solution of common green vitriol in water for about a quarter of an hour or upwards, in contact with an excess of sesquioxide of manganese in fine powder, till the solution affords a white precipitate with ferrocyanide of potassium.

Chloride of manganese may also be formed in a similar manner by boiling an aqueous solution of protochloride of iron with an excess of sesquioxide of manganese, or it may be made with greater facility by dissolving this oxide in the common muriatic acid of commerce, taking care that the oxide be present in excess.

The brown sesquioxide of manganese may be made, not only by means of sulphur, but more readily and better by mixing the common peroxide with about one-third of its weight of peat mould, sawdust or starch, and exposure to a red heat in an open crucible with occasional stirring for about a quarter of an hour, or until the oxide acquires a uniform brown colour.

The sulphate and chloride of manganese being extensively used in dyeing, calico-printing and other arts, and in making the compounds of manganese, the simple means stated of forming those salts, free from iron (it is presumed), are material improvements on the circuitous methods hitherto adopted.

The following was also read:—

Supplement to a paper "On certain Properties of Square Numbers and other Quadratic Forms, with a Table, &c." By Sir Frederick Pollock, F.R.S. &c. Received Jan. 9, 1854.

In the original draft of this paper there was a suggestion that all the terms of the series 1, 3, 7, 13, &c. [there called the Gradation-

Series] possessed the property that was exhibited as belonging to the odd number 197. This was omitted in the copy from some doubt whether it was universally true. Since the paper was read that doubt has been removed, and it turns out that the property belongs not only to *all* the terms of the series 1, 3, 7, 13, &c., but to all odd numbers whatsoever. I am desirous to add to the paper this statement by way of supplement. The property referred to may be thus enunciated:—

Every odd number may be divided into square numbers (not exceeding 4) whose roots (positive or negative) will by their sum or difference [in some form of the roots] give every odd number from 1 to the greatest sum of the roots, which (of course) must always be an odd number.

Or the theorem may be stated in a purely algebraical form, thus:—If there be two equations

$$a^2+b^2+c^2+d^2=2n+1$$
$$a+b+c+d=2r+1,$$

a, b, c, d being each integral or zero, n and r being positive, and r a maximum; then if any positive integer r' (not greater than r) be assumed, it will always be possible to satisfy the pair of equations

$$w^2+x^2+y^2+z^2=2n+1$$
$$w+x+y+z=2r'+1$$

by integral values (positive, negative or zero) of w, x, y, z.

I hope shortly to communicate a proof of the above theorem, independent of any of the usual modes of proving that every odd number is composed of (not exceeding) four square numbers.

Note.—The differences of the roots of 197 were not fully stated in the paper, I add them here:—

197

has 7 forms of roots:—

Forms of roots.	
14, 1, 0, 0	
12, 7, 2, 0	3 ... 7......................17 21
12, 6, 4, 1	1, 3 ...... 9, 11, 13 19, 21, 23,
11, 6, 6, 2	1, 3 ...... 9, 13, 21, 25,
10, 9, 4, 0	3, 5,.................. 15,.............. 23
10, 6, 6, 5	3, 5, 7.............. 15, 17, ... 27.
9, 8, 6, 4	1, 3, 7 11, 15, 19.............. 27.

January 19, 1854.

CHARLES WHEATSTONE, Esq., V.P., in the Chair.

A paper was read, entitled "On the Geometrical Representation of the Expansive Action of Heat, and the Theory of Thermo-dynamic Engines." By William John Macquorn Rankine, Civil Engineer, F.R.S.S. Lond. and Edinb. &c. Received December 5, 1853.

The author remarks, that if abscissæ be measured from an origin